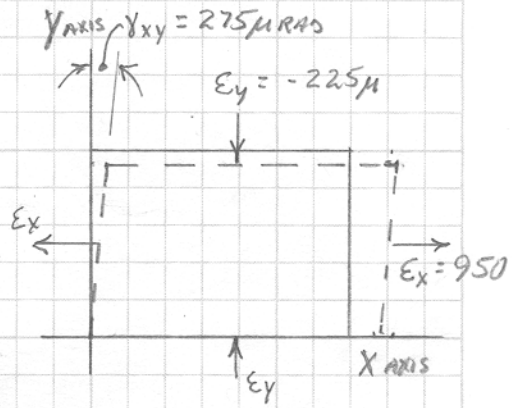
PROBLEM NO. 3-65	MECHANICAL ENGINEERING DEPARTMENT	NAME CDR DREON
DATE 2 FEB 00		SECTION 4322, 6521

Given: STRAIN ON A 2D PLATE

$\varepsilon_x = 950\mu \quad \varepsilon_y = -225\mu$

$\gamma_{xy} = 275\mu$ RADIAN

Find: $\varepsilon_{P_1} \quad \varepsilon_{P_2} \quad \gamma_P \quad \gamma_{MAX} \quad \theta_P$



System Sketch or FBD

Relationships:

$$\varepsilon_{P_1}, \varepsilon_{P_2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \left\{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2\right\}^{1/2}$$

$$\gamma_P = 2\left\{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2\right\}^{1/2} \qquad \tan 2\theta_P = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$R = \left\{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2\right\}^{1/2}$$

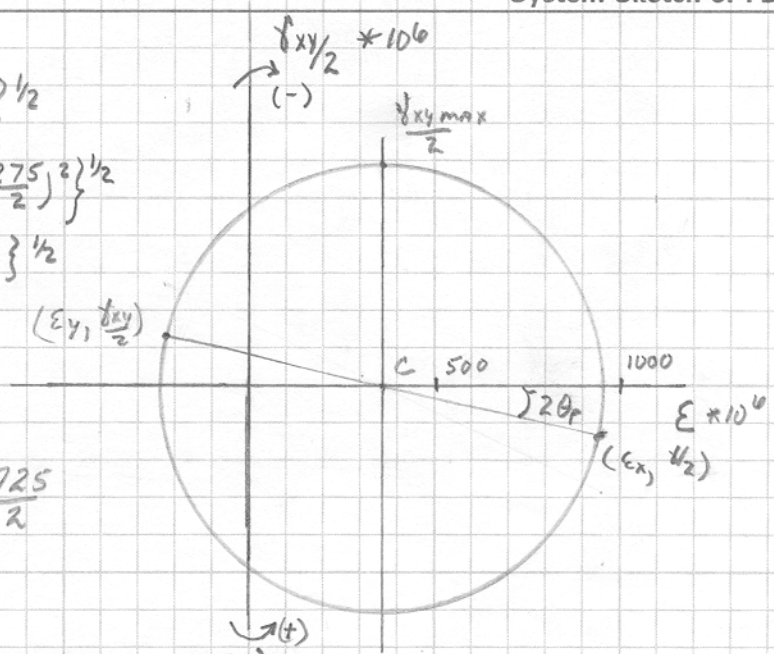
Solution:

$$\text{RADIUS} = \left\{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \frac{\gamma_{xy}}{2}^2\right\}^{1/2}$$

$$= \left\{\left(\frac{950 - (-225)}{2}\right)^2 + \left(\frac{275}{2}\right)^2\right\}^{1/2}$$

$$= \{587.5^2 + 137.5^2\}^{1/2}$$

$$\text{RADIUS} = 603.4\mu$$



$$\text{CENTER} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$= \frac{950 + (-225)}{2} = \frac{725}{2}$$

$$\text{CENTER} = 362.5\mu$$

$$\varepsilon_{P_1}, \varepsilon_{P_2} = \frac{950 - 225}{2} \pm \left\{\left(\frac{950 + 225}{2}\right)^2 + \left(\frac{275}{2}\right)^2\right\}^{1/2}$$

$$= 362.5 \pm 603.4\ \mu$$

$$\varepsilon_{P_1} = 965.9\mu$$

$$\varepsilon_{P_2} = -240.9\mu$$

$$\gamma_P = 2\left\{\left(\frac{950 - (-225)}{2}\right)^2 + \left(\frac{275}{2}\right)^2\right\}^{1/2}$$

$$= 2(603.4)$$

$$\gamma_P = 1206.8\ \mu\text{RAD}$$

$$\tan 2\theta_P = \frac{275}{950 + 225}$$

$$= 275/1175$$

$$\tan 2\theta_P = 0.2340$$

$$\theta_P = \frac{1}{2}\tan^{-1} 0.2340 \qquad \theta_P = 6.59^\circ \circlearrowleft$$

PROBLEM NO. 3-73	MECHANICAL ENGINEERING DEPARTMENT	NAME KOUL
DATE		SECTION

Given: Fig P3-73 $\nu = 0.3$

$\varepsilon_a = 750\mu$ $\theta_a = 0°$

$\varepsilon_b = -125\mu$ $\theta_b = 45°$

$\varepsilon_c = -250\mu$ $\theta_c = 90°$

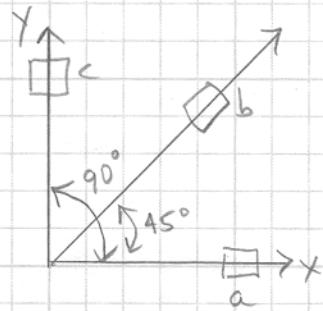
Find: (a) ε_x, ε_y and γ_{xy}

(b) ε_{p1}, ε_{p2}, ε_{p3} and γ_{max}

Relationships:

Eq 3-7a

3-12



System Sketch or FBD

Solution:

(a) $\varepsilon_n = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$

$\theta_a = 0$

$\varepsilon_a = \boxed{\varepsilon_x = 750\mu}$

$\theta_b = 45°$

$$\varepsilon_b = \frac{750}{2} + \frac{\varepsilon_y}{2} + \frac{\gamma_{xy}}{2} = -125\mu$$

$\theta_c = 90°$

$\varepsilon_c = \boxed{\varepsilon_y = -250\mu}$

$$\frac{750}{2} - \frac{250}{2} + \frac{\gamma_{xy}}{2} = -125$$

$$\boxed{\gamma_{xy} = -750\mu}$$

(b) $$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\boxed{\varepsilon_{p1} = 875\mu \qquad \varepsilon_{p2} = -375\mu}$$

$$\varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{0.3}{0.7}(500)$$

$$\boxed{\varepsilon_{p3} = -214\mu}$$

$\varepsilon_{p1} > \varepsilon_{p3} > \varepsilon_{p2}$

so $\gamma_{max} = \gamma_p$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min}$

$\gamma_{max} = 875 + 375$

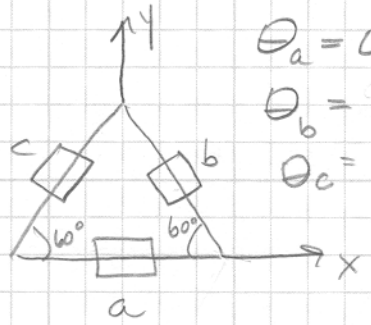
$$\boxed{\gamma_{max} = 1250\mu}$$

PROBLEM NO. 3-78	MECHANICAL ENGINEERING DEPARTMENT	NAME KOUL
DATE		SECTION

Given: $\varepsilon_a = 780\mu$ $\quad \nu = 0.33$

$\varepsilon_b = 315\mu$ $\quad$ rosette shown

$\varepsilon_c = -332\mu$



$\theta_a = 0°$

$\theta_b = 120°$

$\theta_c = 60°$

Find:

(a) $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

(b) $\varepsilon_{p1}, \varepsilon_{p2}, \varepsilon_{p3}, \gamma_{max}$

Relationships:

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

System Sketch or FBD

Solution:

(a) Simultaneously solve three equations above for $\varepsilon_x, \varepsilon_y, \varepsilon_z$

$$\boxed{\begin{aligned} \varepsilon_x &= 780\mu \\ \varepsilon_y &= -251.3\mu \\ \gamma_{xy} &= -781.8\mu \end{aligned}}$$

$$\varepsilon_z = \varepsilon_{p3} = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

$$\varepsilon_z = \varepsilon_{p3} = -\frac{0.33}{1-0.33}(780 - 251.3)$$

$$\boxed{\varepsilon_z = \varepsilon_{p3} = -260\mu}$$

(b) $$\varepsilon_{p1}, \varepsilon_{p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$264.35 \pm 647.1$$

$$\boxed{\varepsilon_{p1} = 911.4\mu} \qquad \boxed{\varepsilon_{p2} = -382.7\mu}$$

$$\gamma_p = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\boxed{\gamma_p = 1294.1\mu}$$

$$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = \varepsilon_{p1} - \varepsilon_{p2} = \tau_p$$

$$\boxed{\gamma_{max} = 1294.1\mu}$$