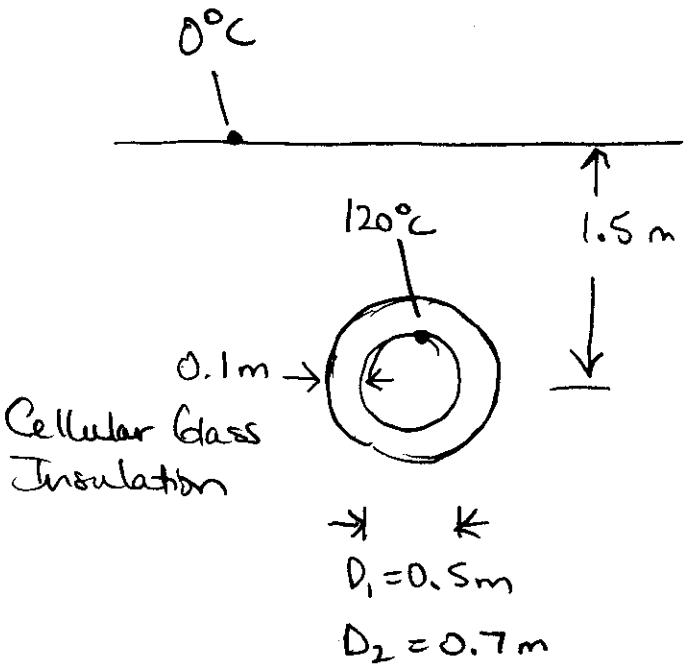
Problem 4.15 Crude Oil Pipeline



Find heat loss per unit length of pipe, q' ?

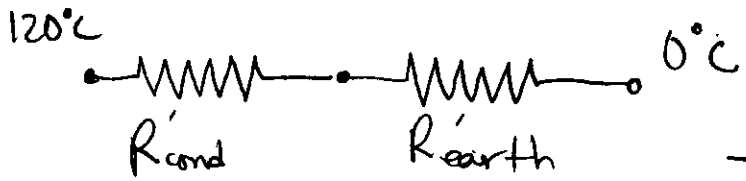


Table A.3

$k_{ins} = 0.058$

$$R'_{cond} = \frac{\ln\left(0.35\text{m}/0.25\text{m}\right)}{2\pi k}$$

Table A.3

$k_{soil} = 0.52$ W/mK

$$R'_{cond} = 0.92 \ \frac{\text{mK}}{\text{W}}$$

$$R'_{earth} = \frac{1}{Sk} \cdot L = \frac{\cosh^{-1}(2z/D)}{2\pi \cancel{L} k} \cdot \cancel{L}$$

Shape Factor

$D_2 = 0.7$ m

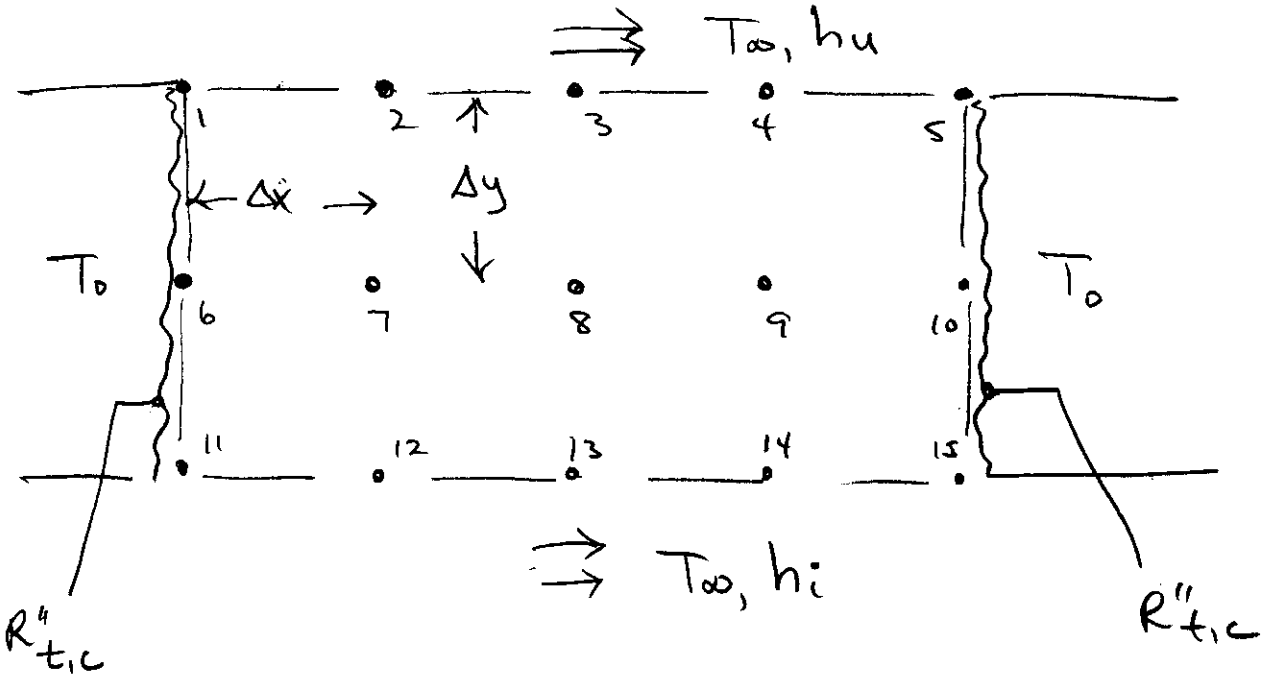
$$R'_{earth} = 0.653 \ \frac{\text{mK}}{\text{W}}$$

$$R'_{TOT} = 1.573 \ \frac{\text{mK}}{\text{W}}$$

$$q' = \frac{\Delta T}{R'} = \boxed{76.3 \ \frac{\text{W}}{\text{m}}}$$

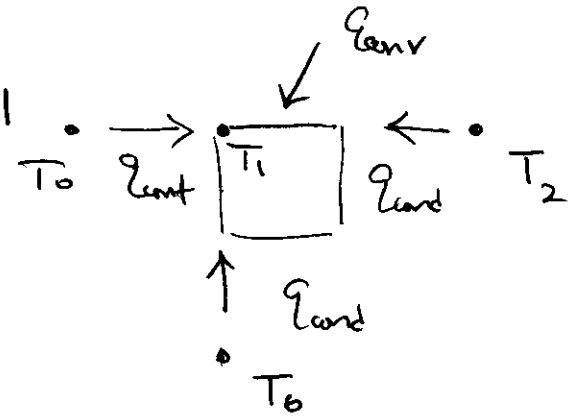
Problem 4.40

Bus Bar



$h_u \neq h_i \qquad \dot{q} \neq 0$

Node 1



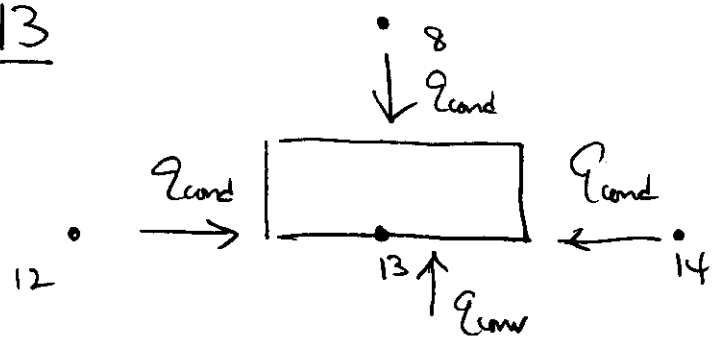
$$q_{cond_{2\to 1}} + q_{cond_{6\to 1}} + q_{contact} + q_{conv} + q_{gen}$$

$$K\frac{\Delta y}{2}\left[\frac{T_2 - T_1}{\Delta x}\right] + K\frac{\Delta x}{2}\left[\frac{T_6 - T_1}{\Delta y}\right]$$

$$+ \frac{\Delta y}{2}\left[\frac{T_0 - T_1}{R''_{t,c}}\right] + h_u\frac{\Delta x}{2}\left[T_\infty - T_1\right] + \dot{q}\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

$$2T_2 + 2T_6 - 4T_1 + \frac{2\Delta y}{K R''_{t,c}}\left[T_0 - T_1\right] + \frac{2h\Delta x}{K}\left[T_\infty - T_1\right] + \frac{\dot{q}\Delta x^2}{K} = 0$$

Node 13



$$q_{cond\,14\to13} + q_{cond\,12\to13} + q_{cond\,8\to13} + q_{conv} + q_{gen} =$$

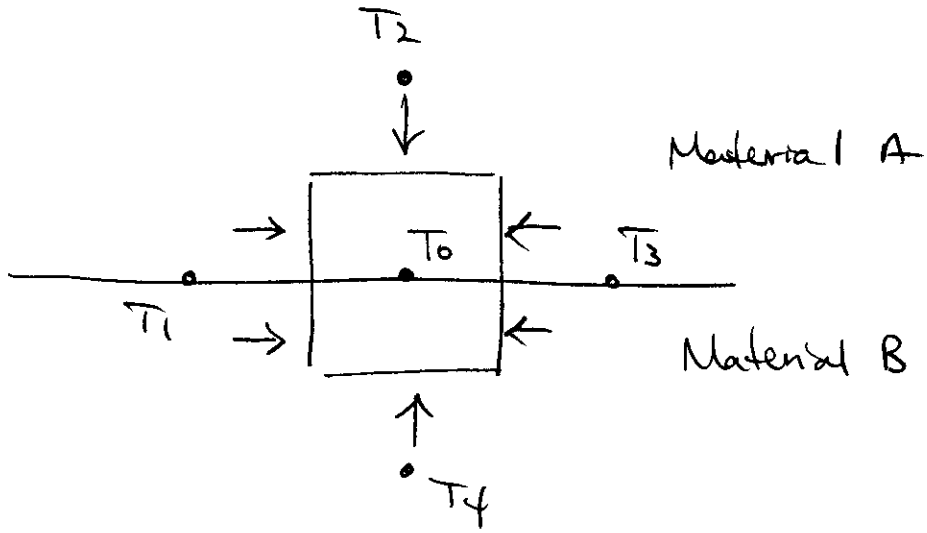
$$K\frac{\Delta y}{2}\left[\frac{T_{14}-T_{13}}{\Delta x}\right] + K\frac{\Delta y}{2}\left[\frac{T_{12}-T_{13}}{\Delta x}\right]$$

$$+ K\Delta x\left[\frac{T_8-T_{13}}{\Delta y}\right] + h\Delta x\left[T_\infty - T_{13}\right]$$

$$+ \dot{q}\,\Delta x\,\frac{\Delta y}{2} = 0$$

$$\boxed{T_{14} + T_{12} + 2T_8 - 4T_{13} + \frac{2h\Delta x}{K}\left[T_\infty - T_{13}\right] + \frac{\dot{q}\,\Delta x^2}{K} = 0}$$

Problem 4.42



$$q_{cond\,2\to 0} + q_{A\,cond\,1\to 0} + q_{B\,cond\,1\to 0} +$$

$$q_{cond\,4\to 0} + q_{A\,cond\,3\to 0} + q_{B\,cond\,3\to 0} = 0$$

$$K_A \cancel{\Delta x}\left[\frac{T_2 - T_0}{\cancel{\Delta y}}\right] + K_A \frac{\cancel{\Delta y}}{2}\left[\frac{T_1 - T_0}{\cancel{\Delta x}}\right]$$

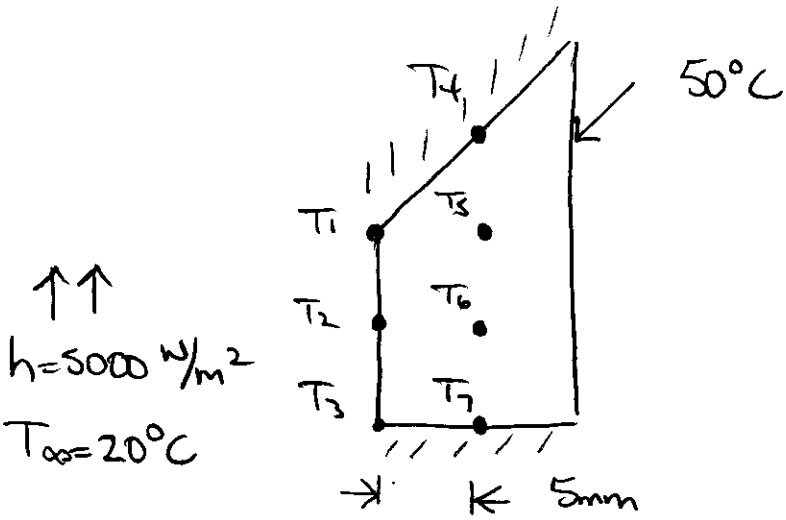
$$+ K_B \frac{\cancel{\Delta y}}{2}\left[\frac{T_1 - T_0}{\cancel{\Delta x}}\right] + K_B \cancel{\Delta x}\left[\frac{T_4 - T_0}{\cancel{\Delta y}}\right]$$

$$+ K_A \frac{\cancel{\Delta y}}{2}\left[\frac{T_3 - T_0}{\cancel{\Delta x}}\right] + K_B \frac{\cancel{\Delta y}}{2}\left[\frac{T_3 - T_0}{\cancel{\Delta x}}\right]$$

$$2K_A T_2 + (K_A + K_B) T_1 + 2K_B T_4$$

$$+ (K_A + K_B) T_3 - 4(K_A + K_B) T_0 = 0$$

Problem 4.49 a) Aluminum Heat Sink



Nodes 5 & 6 – Interior Nodes $\dot{q} = 0$

$$T_1 + T_4 + 50 + T_6 - 4T_5 = 0$$

$$T_2 + T_5 + 50 + T_7 - 4T_6 = 0$$

Node 7 – Half Node Insulated

$$T_3 + 2T_6 + 50 - 4T_7 = 0$$

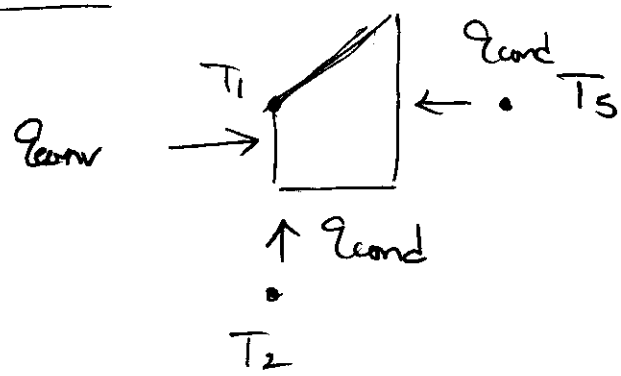
Node 2 – Half Node w/ Convection

$$T_1 + 2T_6 + T_3 - 4T_2 - 2\frac{h\Delta x}{K}T_2 = -2\frac{h\Delta x}{K}T_\infty$$

Node 3 – Quarter Node w/ Convection

$$2T_2 + 2T_7 - 4T_3 - 2\frac{h\Delta x}{K}T_3 = -2\frac{h\Delta x}{K}T_\infty$$

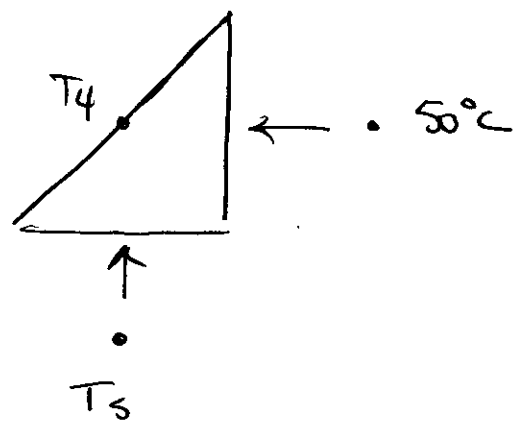
Node 1



$$K\Delta y\left[\frac{T_5-T_1}{\Delta x}\right]+K\frac{\Delta x}{2}\left[\frac{T_2-T_1}{\Delta y}\right]+h\frac{\Delta y}{2}\left[T_\infty-T_1\right]=0$$

$$\boxed{4T_5+2T_2-6T_1-\frac{2h\Delta x}{K}T_1=-\frac{2h\Delta x}{K}T_\infty}$$

Node 4



$$K\Delta y\left[\frac{50-T_4}{\Delta x}\right]+K\frac{\Delta x}{\Delta y}\left[T_5-T_4\right]=0$$

$$\boxed{2T_5+2\cdot 50-4T_4=0}$$

```
k = 237
h=5000
dx =0.005
Bi = h*dx/k
Tinf = 20

"Node 1"
4*T[5] + 2*T[2] - 6*T[1] - 2*Bi*T[1] = -2*Bi*Tinf

"Node 2"
T[1] + 2* T[6] + T[3] - 4*T[2] - 2*Bi*T[2] = -2*Bi*Tinf

"Node 3"
2*T[2] + 2*T[7] - 4*T[3] - 2*Bi*T[3] = -2*Bi*Tinf

"Node 4"
2*T[5] +100 - 4*T[4] = 0

"Node 5"
T[1] + T[4] + 50 + T[6] - 4*T[5] = 0

"Node 6"
T[2] + T[5] + 50 + T[7] - 4*T[6] = 0

"Node 7"
T[3] + 2*T[6] + 50 - 4*T[7] = 0

q_prime/8 = h*dx/2*(T[1] - Tinf) + h*dx*(T[2] - Tinf) + h*dx/2*(T[3] - Tinf)
```

SOLUTION
Unit Settings: [F]/[psia]/[lbm]/[degrees]

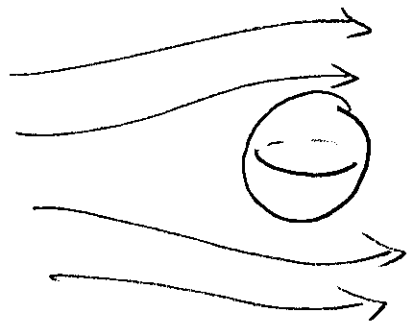
Bi = 0.1055	dx = 0.005	h = 5000
k = 237	q_{prime} = 10321	Tinf = 20

Arrays Table

	T_i
1	46.57
2	45.63
3	45.39
4	49.22
5	48.44
6	47.98
7	47.84

Problem 5.6

Sphere (pure Copper)



$D = 12.7\ mm$ $\quad K = 401\ W/mK$

$T_i = 66°C$ $\quad \rho = 8933\ kg/m^3$

$T_\infty = 27°C$ $\quad C_p = 385\ J/kg\,K$

$T = 55°C$ at $t = 69$ second

Determine the heat transfer coefficient.

$$\frac{\theta}{\theta_i} = \exp\left[-Bi \cdot Fo\right] \qquad Fo = \frac{\alpha t}{L_c^2} \qquad \alpha = \frac{K}{\rho C_p}$$

$$L_c = r/3 = 0.00212\ m$$

$$\frac{\theta}{\theta_i} = \frac{55-27}{66-27} = 0.718 \qquad Fo = 1790$$

$$Bi = 1.851 \times 10^{-4} = \frac{h L_c}{K}$$

$$\boxed{h = 35\ W/m^2K}$$

"Problem 5.8"

```
h=10
k=180
T_infinity=T_s
T_s=25+273.15
epsilon=0.8
sigma=5.67e-8
rho=2800
c=900
A_s=0.04
q_h=1.25e4
T_f=135+273.15
L=0.007

T[1]=T_s
time[1] = 0
dt=2
duplicate i=2,100
T[i]=T[i-1]+dt/(rho*c*L)*(q_h+h*(T_s-T[i])+epsilon*sigma*(T_s^4-T[i]^4))
time[i]=dt*(i-1)
end
```

$$\dot{E}_{stored} = q''_h A_s - h A_s [T - T_\infty] - \varepsilon\sigma [T^4 - T_{surr}^4]$$

$$\rho \frac{V C_p}{A_s} \frac{dT}{dt} = q''_h - h [T - T_\infty] - \varepsilon\sigma [T^4 - T_{surr}^4]$$

$$\frac{T[i] - T[i-1]}{\Delta t} = \frac{A_s}{\rho C_p V} \left[q''_h - h(T[i] - T_\infty) - \varepsilon\sigma (T[i]^4 - T_{surr}^4) \right]$$

Δt - time increment

Arrays Table

	T_i [K]	$time_i$
1	298.2	0
2	299.6	2
3	301	4
4	302.4	6
5	303.8	8
6	305.2	10
7	306.6	12
8	308	14
9	309.4	16
10	310.8	18
11	312.2	20
12	313.6	22
13	315	24
14	316.4	26
15	317.7	28
16	319.1	30
17	320.5	32
18	321.9	34
19	323.2	36
20	324.6	38
21	326	40
22	327.4	42
23	328.7	44
24	330.1	46
25	331.4	48
26	332.8	50
27	334.1	52

135°C → 408 K

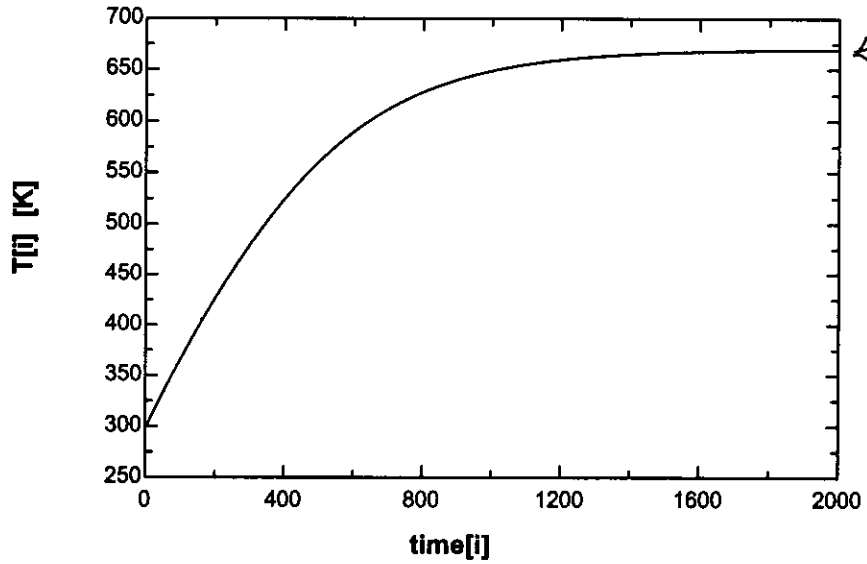
Arrays Table

	T_i [K]	$time_i$
28	335.5	54
29	336.8	56
30	338.2	58
31	339.5	60
32	340.9	62
33	342.2	64
34	343.5	66
35	344.9	68
36	346.2	70
37	347.5	72
38	348.9	74
39	350.2	76
40	351.5	78
41	352.8	80
42	354.1	82
43	355.4	84
44	356.7	86
45	358	88
46	359.3	90
47	360.6	92
48	361.9	94
49	363.2	96
50	364.5	98
51	365.8	100
52	367.1	102
53	368.4	104
54	369.7	106
55	370.9	108
56	372.2	110
57	373.5	112
58	374.8	114
59	376	116
60	377.3	118
61	378.6	120
62	379.8	122
63	381.1	124
64	382.3	126
65	383.6	128
66	384.8	130
67	386.1	132
68	387.3	134
69	388.5	136
70	389.8	138
71	391	140
72	392.2	142
73	393.5	144
74	394.7	146
75	395.9	148
76	397.1	150

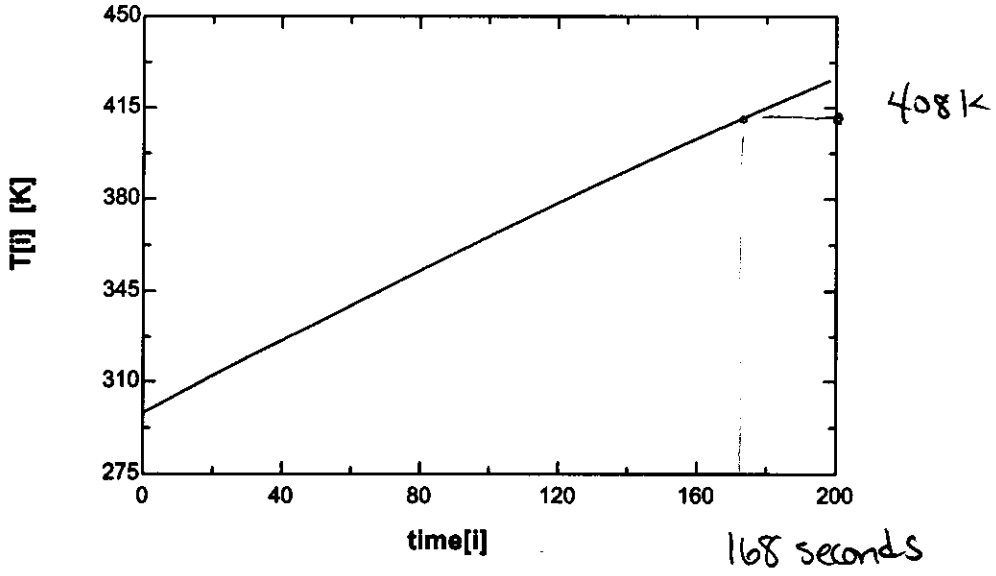
Arrays Table

	T_i [K]	$time_i$
77	398.3	152
78	399.6	154
79	400.8	156
80	402	158
81	403.2	160
82	404.4	162
83	405.6	164
84	406.8	166
85	408	168
86	409.1	170
87	410.3	172
88	411.5	174
89	412.7	176
90	413.9	178
91	415	180
92	416.2	182
93	417.4	184
94	418.5	186
95	419.7	188
96	420.8	190
97	422	192
98	423.2	194
99	424.3	196
100	425.4	198

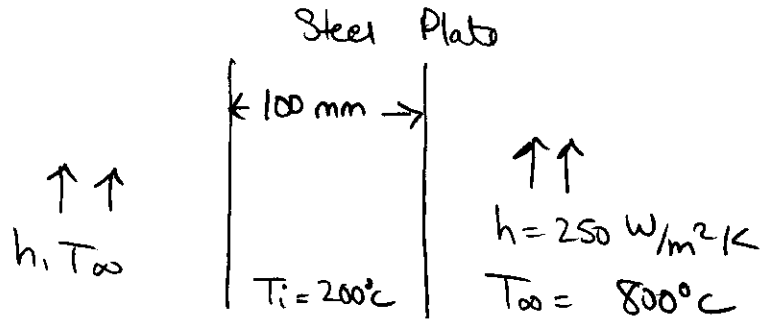
→ time = 168 seconds



← Steady State


450
415
380
345
310
275
T[i] [K]
0
40
80
120
160
200
time[i]
408K
168 seconds

Problem 5.34 Annealing Process



$K = 48\ W/mK$

$\rho = 7830\ kg/m^3$

$C_p = 550\ J/kgK$

How long should remain in the furnace for the minumum temperature to reach 550°C

- Minimum Temp will be at the middle of the plate

$$\frac{\theta_o}{\theta_i} = \frac{550 - 800}{200 - 800} = 0.4167$$

$$Bi = \frac{hL}{K} = 0.26 \qquad L = 0.05m$$

$$Bi^{-1} = 3.84$$

Figure D.1

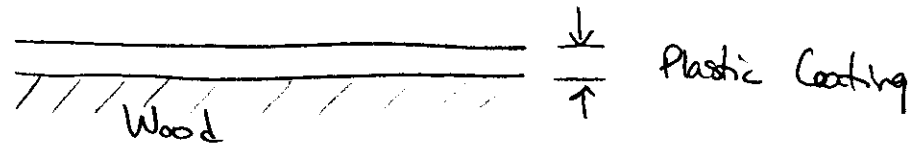
$$Fo = 3.8 = \frac{\alpha t}{L^2}$$

$$\alpha = \frac{k}{\rho C_p}$$

$$\alpha = 11.1 \times 10^{-6}\ m^2/s$$

$$\boxed{t = 852\ s = 14.2\ min}$$

Problem 5.45



$K = 0.25\ W/mK$

$L = 0.002\ m$ $\quad h = 200\ W/m^2K$ $\quad \alpha = 1.2 \times 10^{-7}\ m^2/s$

$T_i = 200°C$ $\quad T_\infty = 25°C$

<u>How long</u> <u>will</u> it take for surface to reach 42°C

$$Bi = \frac{hL}{K} = 1.6 \rightarrow Bi^{-1} = 0.6250$$

at surface

$$\frac{x}{L} = 1 \qquad \frac{\theta}{\theta_o} = 0.52$$

Figure D.2

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = 0.0971$$

$$\frac{\theta_o}{\theta_i} = 0.1867$$

Figure D.1

$$F_o = 1.8 = \frac{\alpha t}{L^2} \Rightarrow t = 60\ s$$

$$\frac{\theta_o}{\theta_i} = 0.1091 = \frac{T_o - T_\infty}{T_i - T_\infty} \Rightarrow T_o = 57.7$$