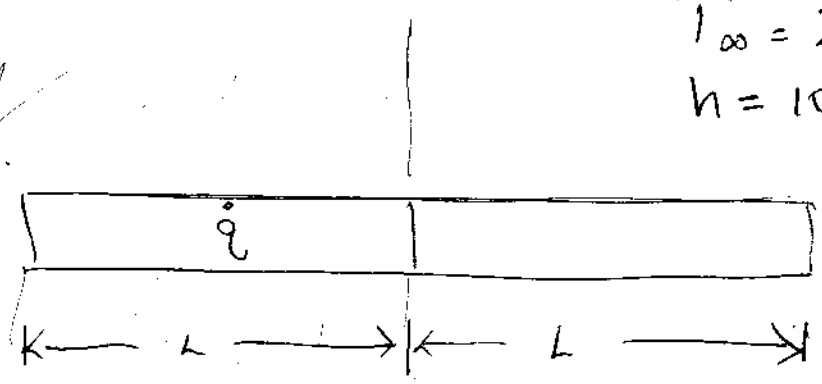
Problem 3.112

$T_\infty = 20°C$

$h = 100\ W/m^2K$



$L = 50\ mm$

$D = 5\ mm$

$K = 25\ W/mK$

$\dot{q} = 1 \times 10^6\ W/m^3$

a) Find T_b assuming a very long fin

$$q_f = \sqrt{hPkA_c}\,(T_b - T_\infty)$$

$$q_f = \dot{q} \cdot \frac{\pi D^2}{4} \cdot L$$

$$\dot{q}\frac{\pi D^2}{4} \cdot L = \sqrt{h \pi D k \frac{\pi D^2}{4}}\,(T_b - T_\infty)$$

$$T_b = \dot{q} \cdot L \sqrt{\frac{D}{4hK}} + T_\infty$$

b) Inside embedded portion of rod

$$T(x) = -\dot{q}\frac{x^2}{2K} + C_1 x + C_2$$

$\left.\frac{dT}{dx}\right|_{x=0} = 0$ Insulated on the left

$T|_{x=L} = T_b$ At the base of the fin

$$\frac{dT}{dx} = -\dot{q}\frac{x}{K} + C_1 \qquad \left.\frac{dT}{dx}\right|_{x=0} = C_1 = 0$$

$$T\Big|_{x=L} = -\frac{\dot{q}L^2}{2K} + C_2 = T_b$$

$$C_2 = T_b + \frac{\dot{q}L^2}{2K}$$

$$T(x) = -\frac{\dot{q}x^2}{2K} + \frac{\dot{q}L^2}{2K} + T_b$$

$$T(0) = \frac{\dot{q}L^2}{2K} + T_b$$

c) Plot the temperature distribution

[See attached EES File]

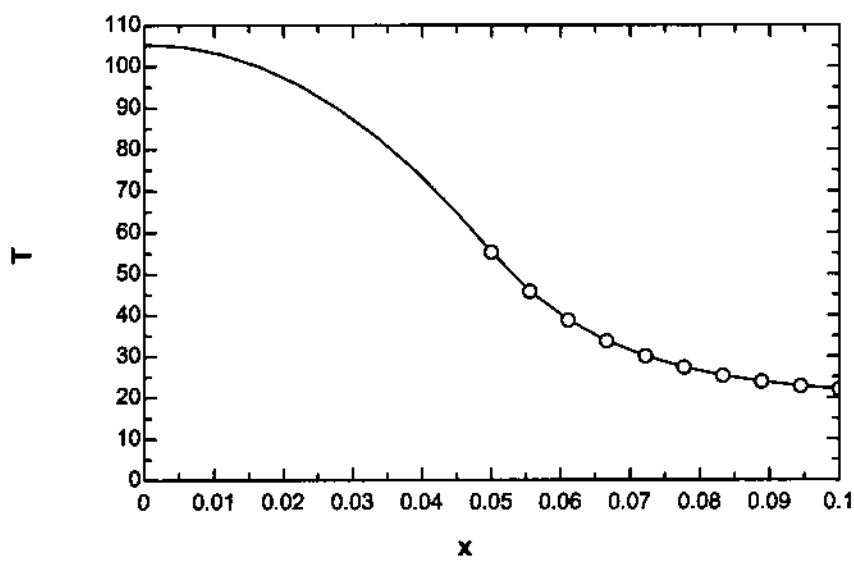
"Problem 3.112 part (c)"

```
q_dot = 1e6
h = 100
L= 0.05
D = 0.005
k = 25
Tinf = 20

T = -q_dot*x^2/(2*k) + q_dot*L^2/(2*k) + Tb

Tb = q_dot*L*sqrt(D/(4*h*k)) + Tinf

m = sqrt((4*h)/(k*D))
(Tf - Tinf) / (Tb - Tinf) = exp(-m*(x2-L))
```

Parametric Table: Table 1

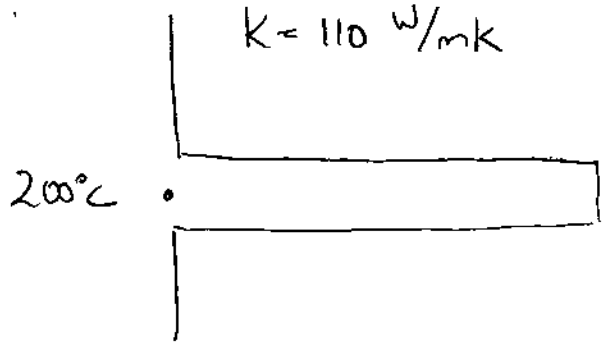
	x	x2	T	Tf
Run 1	0	0.05	105.4	55.36
Run 2	0.005556	0.05556	104.7	45.82
Run 3	0.01111	0.06111	102.9	38.86
Run 4	0.01667	0.06667	99.8	33.77
Run 5	0.02222	0.07222	95.48	30.06
Run 6	0.02778	0.07778	89.92	27.35
Run 7	0.03333	0.08333	83.13	25.36
Run 8	0.03889	0.08889	75.11	23.92
Run 9	0.04444	0.09444	65.85	22.86
Run 10	0.05	0.1	55.36	22.09

at the end of the fin $T|_{x=L} \neq T_\infty$ ∴ not infinite



Problem 3.120 Brass Rod

$L = 100$ mm $D = 5$mm



$T_\infty = 20°C$

$h = 30\ W/m^2K$

What is the temperature of rod at 25, 50 and 100 mm

$\theta_b = 180°C$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh mL}$$

$m = \sqrt{\frac{hP}{KA_c}} = 14.77$ $L = 100$ mm

$X = 25$ mm $\frac{\theta}{\theta_b} = 0.7271$ $\theta = 130.9°C$

$\boxed{@\ x = 25mm\ T = 150.9°C}$

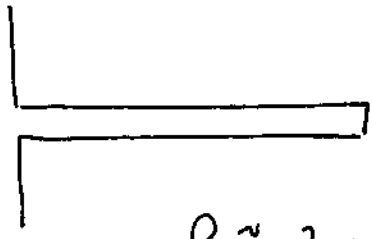
$X = 50$ mm $\frac{\theta}{\theta_b} = 0.555$ $\theta = 99.9°C$

$\boxed{@\ x = 50\ mm\ T = 119.9°C}$

$X = 100$mm $\frac{\theta}{\theta_b} = 0.427$ $\theta = 76.8°C$

$\boxed{@\ x = 100\ mm\ T = 96.8°C}$

" Problem 3.121 - Influence of Tip Conditions on Fin Performance "



$L_f = 10$ mm

$t = 1$ mm

$w >> t$

$P \cong 2w$

$A_c = wt$

$m = \sqrt{\frac{hP}{kA_c}}$

$m = \sqrt{\frac{2h}{kt}}$

```
"Given"
k=180
L=0.01
t = 0.001
"w>>t"
Tb = 100
Tinf = 25
h = 100

m = sqrt((2*h)/(k*t))
Theta_b = Tb-Tinf
Mp = sqrt(2*h*t*k)*Theta_b

"Find qf', Rf', efficiency and effectiveness for Case A & B and compare to and infinite fin"

"Case A"
 qfp_A = Mp*(sinh(m*L)+h/(m*k)*cosh(m*L))/(cosh(m*L)+h/(m*k)*sinh(m*L))
Rfp_A = Theta_b/qfp_A
eff_A = qfp_A/ (h*t*Theta_b)
eta_A = qfp_A / (h*(2*L+t)*Theta_b)

"Case B"
 qfp_B = Mp*tanh(m*L)
Rfp_B = Theta_b/qfp_B
eff_B = qfp_B/ (h*t*Theta_b)
eta_B = qfp_B / (h*(2*L)*Theta_b)

"Infinite"
 qfp_I = Mp
Rfp_I = Theta_b/qfp_I
eff_I = qfp_I/ (h*t*Theta_b)
eta_I = 0
```

Since w is unknown the answers are reported per unit width

$q_f' = \frac{q_f}{w}$

$R_f' = R_f \cdot w$

SOLUTION

Unit Settings: [F]/[psia]/[lbm]/[degrees]

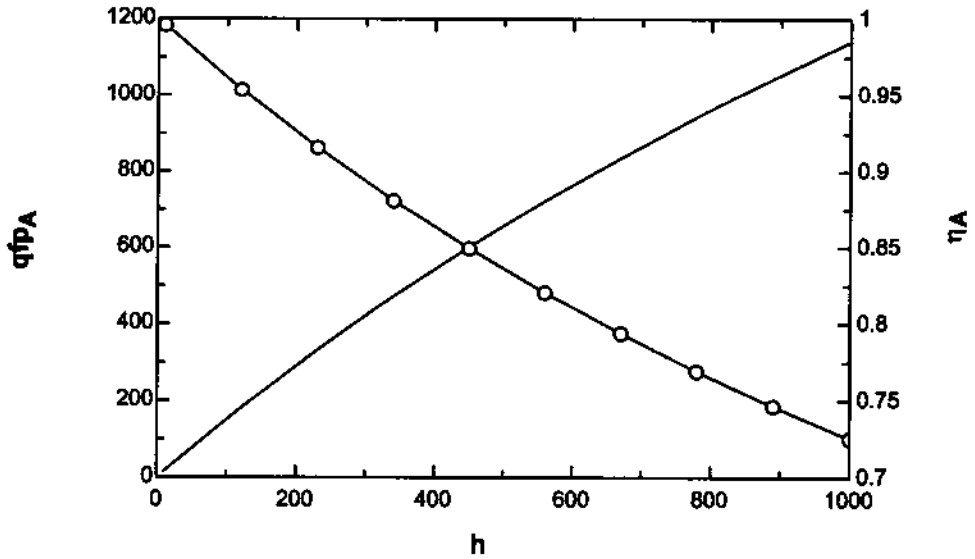
$eff_A = 20.18$	$eff_B = 19.29$	$eff_I = 60$	$\eta_A = 0.9611$
$\eta_B = 0.9645$	$\eta_I = 0$	h = 100	k = 180
L = 0.01	m = 33.33	Mp = 450	$qfp_A = 151.4$
$qfp_B = 144.7$	$qfp_I = 450$	$Rfp_A = 0.4955$	$Rfp_B = 0.5184$
$Rfp_I = 0.1667$	t = 0.001	Tb = 100	$\theta_b = 75$
Tinf = 25			

Parametric Table: Table 1

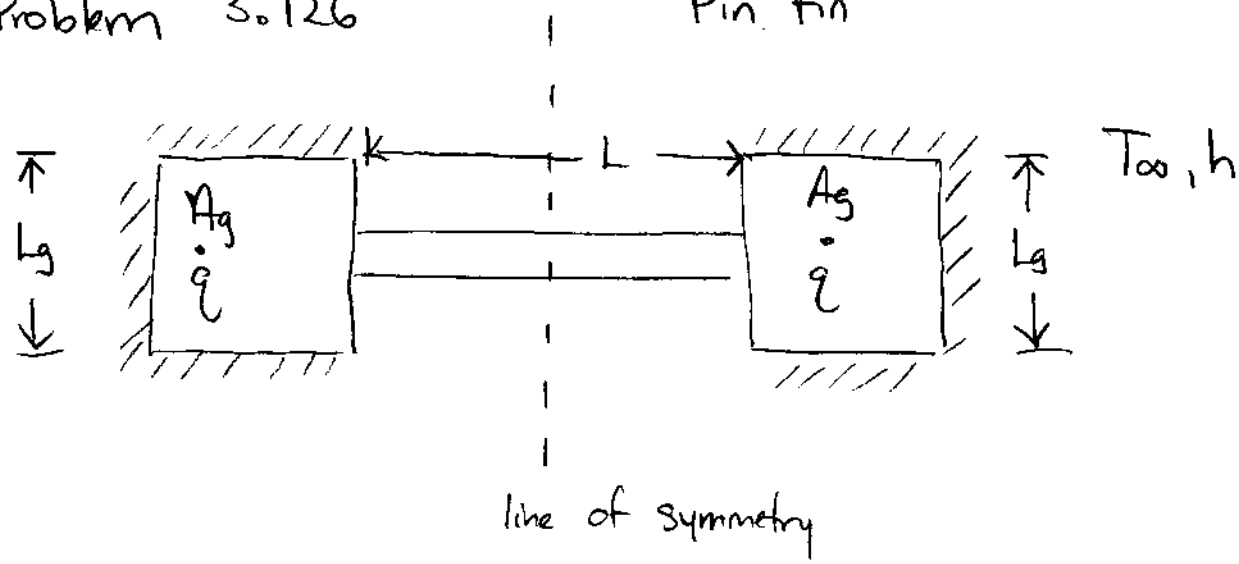
	h	qfp_A	η_A
Run 1	10	15.69	0.9959
Run 2	120	180.3	0.9537
Run 3	230	331.7	0.9156
Run 4	340	471.8	0.881
Run 5	450	602	0.8494
Run 6	560	723.6	0.8205

Parametric Table: Table 1

	h	qfp_A	η_A
Run 7	670	837.7	0.7939
Run 8	780	945.1	0.7693
Run 9	890	1047	0.7466
Run 10	1000	1143	0.7255



Problem 3.126 Pin Fin



- Fin with adiabatic tip of length $L/2$
- Convection for area not covered by the fin
- $q_f + q_b = q_{out} = q_{gen}$

$$A_c = \frac{\pi D^2}{4}$$

$$q_{gen} = \dot{q} \cdot A_g \cdot L_g$$

$$P = \pi D$$

$$q_b = h \cdot (A_g - A_c)(T_b - T_\infty)$$

$$q_f = \sqrt{hPkA_c}\,\tanh\left(\sqrt{\frac{hP}{kA_c}}\,L/2\right)(T_b - T_\infty)$$

$$\dot{q} A_g L_g = \left[h\left(A_g - \frac{\pi D^2}{4}\right) + \sqrt{\frac{hk\pi^2 D^3}{4}}\,\tanh\left(\sqrt{\frac{4h}{kD}}\,\frac{L}{2}\right)\right](T_b - T_\infty)$$

$$T_b = \frac{\dot{q} A_g L_g}{\left[h\left(A_g - \frac{\pi D^2}{4}\right) + \sqrt{\frac{hk\pi^2 D^3}{4}}\,\tanh\left(\sqrt{\frac{4h}{kD}}\,\frac{L}{2}\right)\right]} + T_\infty$$

"Problem 3.136 - Performance Logic Chip Heat Sink "

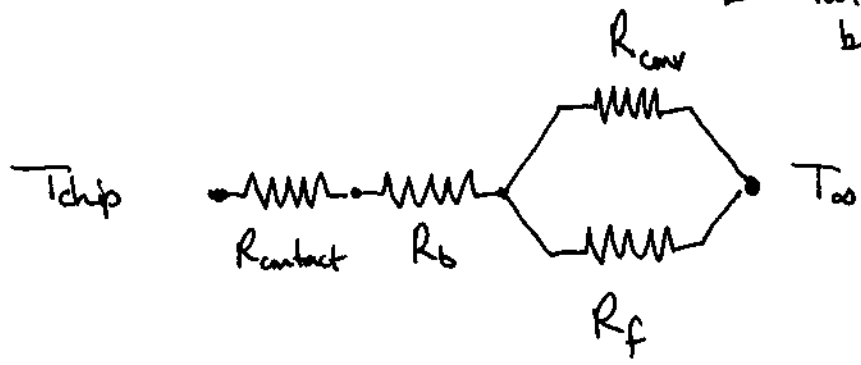
```
Tinf = 25
h = 1500
k = 400
Wc = 0.016
w = 0.00025
S = 0.0005
Lf = 0.006
Lb = 0.003
P = 4*w
Ac = w^2
N*(S) = Wc
Nf = N^2
```

Need to Find the Number of Fins

```
Af = w^2 * Nf
Atot = Wc^2
Awof = Atot - Af

Rb = Lb/(k*Atot)

Rcpp = 5e-6
Rcontact = Rcpp/Atot

Rwof = 1/(h*Awof)

m = sqrt((h*P)/(k*Ac))
Rf = (sqrt(h*P*k*Ac)*(sinh(m*Lf)+h/(m*K)*cosh(m*Lf))/(cosh(m*Lf)+h/(m*K)*sinh(m*Lf)))^(-1) / Nf

Rtot = Rcontact + Rb + ( (1/Rf) + 1/Rwof )^(-1)

Tchip = 85
q = (Tchip - Tinf) / Rtot
```

← Max Tchip ⇒ max q

from area not covered by fins

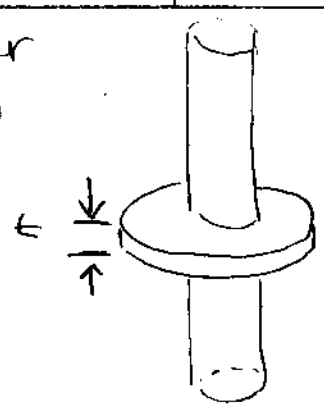


SOLUTION
Unit Settings: [F]/[psia]/[lbm]/[degrees]

Ac = 6.250E-08	Af = 0.000064	Atot = 0.000256
Awof = 0.000192	h = 1500	k = 400
Lb = 0.003	Lf = 0.006	m = 244.9
N = 32	Nf = 1024	P = 0.001
q = 276.5	Rb = 0.0293	Rcontact = 0.01953
Rcpp = 0.000005	Rf = 0.1767	Rtot = 0.217
Rwof = 3.472	S = 0.0005	Tchip = 85
Tinf = 25	w = 0.00025	Wc = 0.016

Problem 3.145

Annular Fin



Aluminum
$K = 237\ W/mK$

$r_1 = 0.025\ m$

$T_s = T_b = 200°C$

$t = 0.004\ m$

$L = 0.015\ m$

$T_\infty = 20°C$

$h = 40\ W/m^2K$

a) Find η_f and ε_f — Figure 3.19

$L_c = L + t/2 = 0.017\ m$

$A_p = L_c t = 6.8 \times 10^{-5}$

$L_c^{3/2}\left(\frac{h}{KA_p}\right)^{1/2} = 0.11$

$r_2 = 0.040$

$r_{c_2} = r_2 + t/2 = 0.042$

$\frac{r_{c_2}}{r_1} = 1.68$

$r_1 = 0.025\ m$

$\eta_f = 0.95$

$A_f = 2\pi(r_{2c}^2 - r_1^2)$

$A_f = 7.156 \times 10^{-3}\ m^2$

$\theta_b = 180°C$

$q_f = \eta_f \cdot h A_f \theta_b = 48.9\ W$

b) 125 fins per meter find q'

$q'_f = 125 \cdot q_f = 6118\ W/m$

$q'_{conv} = h(A_s - 125 \cdot A_b)\theta_b$

$q'_{conv} = 566\ W/m$

$q' = 6684\ W/m$

Base Area of one fin

$A_b = 2\pi r_1 t$

$A_b = 6.28 \times 10^{-4}\ m$

$A_s = 2\pi r_1 \cdot 1m$

$A_s = 0.157\ m^2$