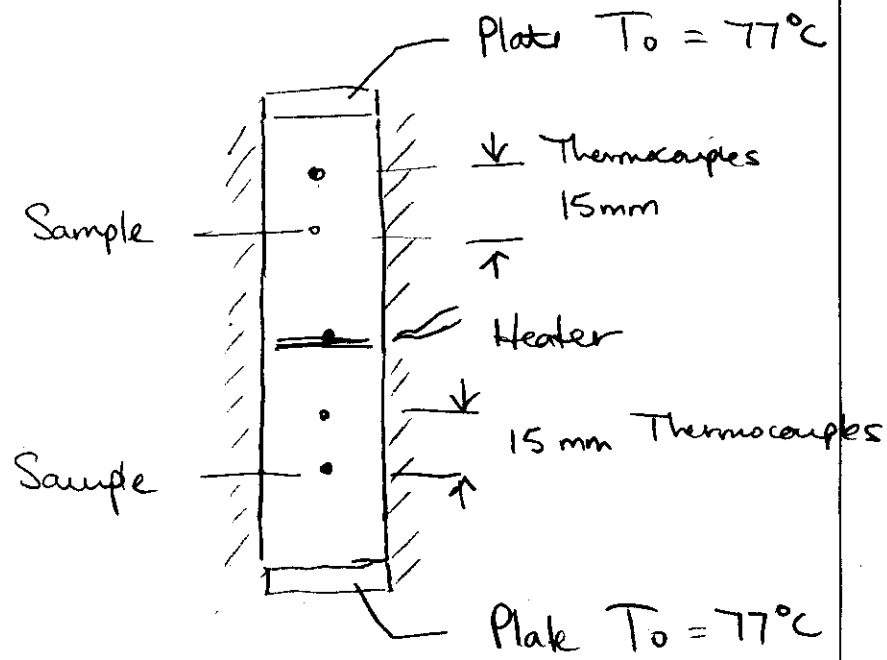
Problem 2.17



Samples

$D = 30mm$

$L = 60mm$

$A_c = \pi \frac{D^2}{4}$

$A_c = 7.069 \times 10^{-4} \, m^2$

a) Two samples SS316

$P = V \cdot I = 100 V \cdot 0.353 A = 35.3 W$

$\Delta T_1 = \Delta T_2 = 25°C$ $\quad q = \frac{P}{2}$ ← half total heat goes through each sample

$q = -k A_c \frac{dT}{dx} \Rightarrow k = \frac{q}{A \frac{\Delta T}{\Delta L}} = \frac{17.65 W}{7.069 \times 10^{-4} m^2 \cdot \frac{25K}{0.015}}$

$k = 14.98$

Average Temp – (find T_H – temp at heater)

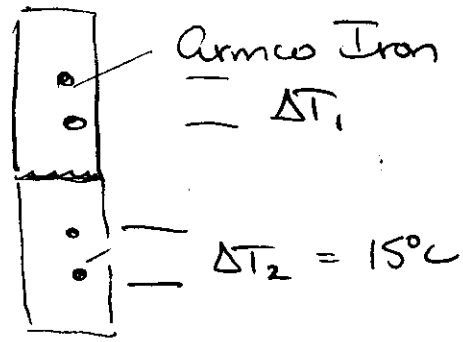
$q = k A_c \left[\frac{T_H - T_0}{L}\right]$ $\quad L = 0.06 m$, $T_0 = 77°C$

$T_H = \frac{q \cdot L}{k A_c} + T_0 = 177°C$

$T_{average} = 127 °C$

b) Replace upper sample

$P = 60.1\ W$



$$q = q_{iron} + q_{SS316}$$

$$q_{SS} = K_{SS} A_c \frac{\Delta T_2}{0.015\,m} = 14.98 \frac{W}{mK} \cdot 7.069 \times 10^{-4}\,m^2 \cdot \frac{15°C}{0.015\,m}$$

$$q_{SS} = 10.6\ W$$

$$q_{iron} = 49.5\,W = K_{IRON} A_c \frac{\Delta T_1}{0.015\,m}$$

$$K_{iron} = \frac{49.5\,W \cdot 0.015\,m}{7.069 \times 10^{-4}\,m^2 \cdot 15\,K}$$

$$K_{iron} = 70\ W/mK$$

Find T_H

$$q = K_{iron} \cdot A_c \cdot \left[\frac{T_H - T_0}{L}\right] \qquad L = 60\,mm$$

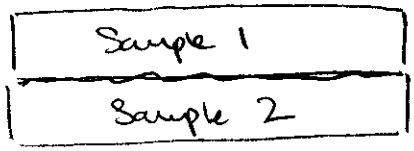
$T_H = 137°C$

$T_{average} = 107°C$

c) - Advantage of two identical samples is that the heat load splits evenly
- Heat leakage would be significant for low conductivity samples
- ΔT_1 always equals ΔT_2

Problem 2.19 Two Identical Samples

$q = 15\,W$



Eq.(1) $T_o(t) - T_i = 2q''\left(\frac{t}{\pi \rho C_p k}\right)^{1/2}$

$T_o(30s) = 24.57°C$

$q'' = \frac{q}{A}$ $\quad A = 2 \cdot A_c = 2\frac{\pi D^2}{4}$

$A = 0.0057\,m^2$

$q'' = 2653\,\frac{W}{m^2}$

<u>k & ρC_p are unknown</u>

use final temp to find ρC_p

↳ after 120s of heating the samples cool to a uniform temperature of 33.5°C

$Q = q \cdot \Delta t = 1800\,J$ $\quad L = 10\,mm$

$Q = \rho C_p V \Delta T$ $\quad V = \frac{\pi D^2}{4} \cdot 2L$

$V = 5.655 \times 10^{-5}\,m^3$

$\rho C_p = \frac{Q}{V \Delta T}$ $\quad \Delta T = 33.5°C - 23°C$

$\Delta T = 10.5°C$

$\rho C_p = 3.0315 \times 10^6\,\frac{J}{m^3 K}$

$\rho = 3965 \ kg/m^3$

$C_p = 764.6 \ J/m^3 K$

* Now K is the only unknown in Eq. (1)

$$\pi \rho \frac{C_p K}{t} = \left(\frac{2q''}{T_o(t) - T_i} \right)^2$$

$$K = \frac{t}{\pi \rho C_p} \left(\frac{2q''}{T_o(t) - T_i} \right)^2$$

$t = 30s$

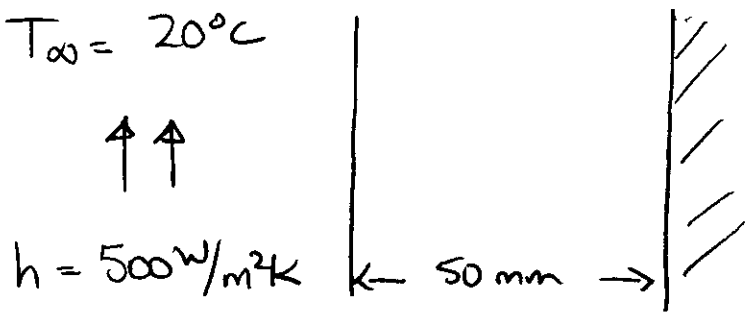
$T_o(30s) = 24.57°C$

$q'' = 2653 \ \frac{W}{m^2}$

$K = 35.98 \ W/mK$

- Polycrystalline Aluminum oxide -

Problem 2.26 a) & b) 1D, Steady State Conduction
$\dot{q} \neq 0$



$k = 5 W/mK$

$$T(x) = a + bx + cx^2$$

$$T(0) = 120°C$$

$$\frac{dT}{dx}(L) = 0 \quad \leftarrow \text{well insulated}$$

a) applying an overall energy balance find $\dot{q}$

$$\dot{q} \cdot V = q_{out}$$

$$\dot{q} \cdot V = q''\Big|_{x=0} \cdot A$$

$$\dot{q} \cdot L = h\left[T\big|_{x=0} - T_\infty\right] \qquad L = 0.05m$$

$$\dot{q} = h\,\frac{\left[T\big|_{x=0} - T_\infty\right]}{L}$$

$$\dot{q} = 1 \times 10^6 \ \frac{W}{m^3}$$

b) Find a, b & c

$$T(0) = a = 120°C$$

$$\frac{dT}{dx}(L) = b + 2c(L) = 0$$

energy leaving left wall

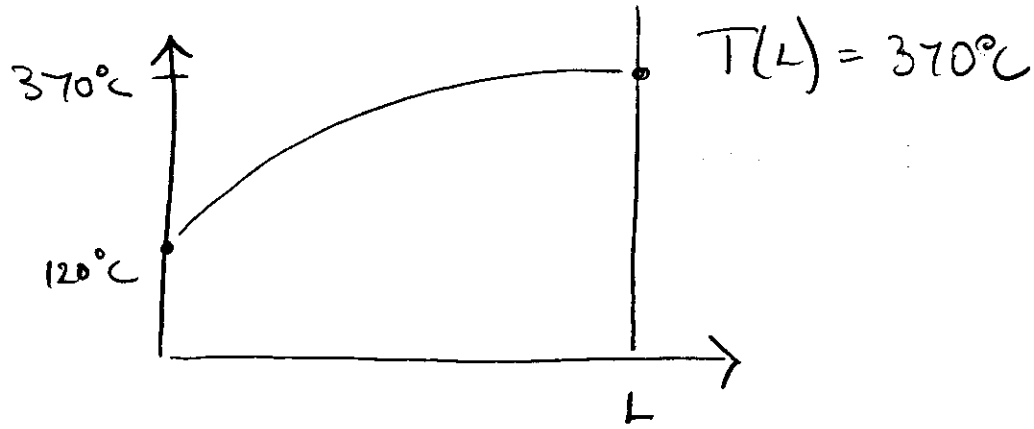
$$k\frac{dT}{dx}(0) = \dot{q} \cdot L$$

$$k\,b = \dot{q} \cdot L$$

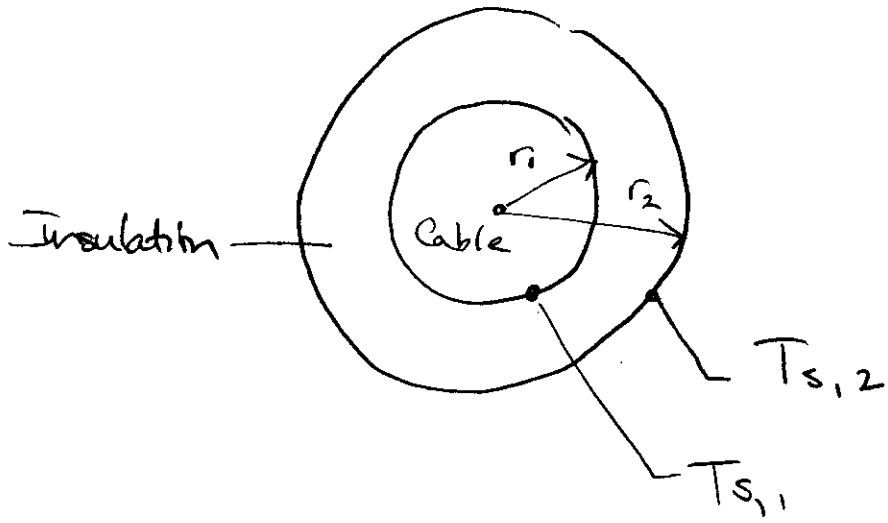
$$b = +\frac{\dot{q} \cdot L}{k} = -10{,}000 \ \frac{K}{m}$$

$$c = -\frac{b}{2L} = 100{,}000 \ K/m^2$$

$$\boxed{T(x) = 120°C + 10{,}000\,x - 100{,}000\,x^2}$$



Problem 2.41 Electrical Cable



$\dot{q}$ – resistive heating in the cable

a) write steady state Heat Diffusion Equation

$$\rho C_p \cancel{\frac{\partial T}{\partial t}} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\cancel{\left(k\frac{\partial T}{\partial \phi}\right)} + \frac{\partial}{\partial z}\cancel{\left(k\frac{\partial T}{\partial z}\right)} + \dot{q}$$

$$\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = -\dot{q}r \quad \Rightarrow \text{Cable}$$

$$\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0 \quad \Rightarrow \text{Insulation}$$

Verify $T(r) = T_{s,2} + (T_{s,1} - T_{s,2})\dfrac{\ln(r/r_2)}{\ln(r_1/r_2)}$

$$\frac{dT}{dr} = (T_{s_1} - T_{s_2}) \cdot \frac{1}{r \ln(r_1/r_2)}$$

$$k r \frac{dT}{dr} = \frac{k(T_{s_1} - T_{s_2})}{\ln(r_1/r_2)} = \text{constant}$$

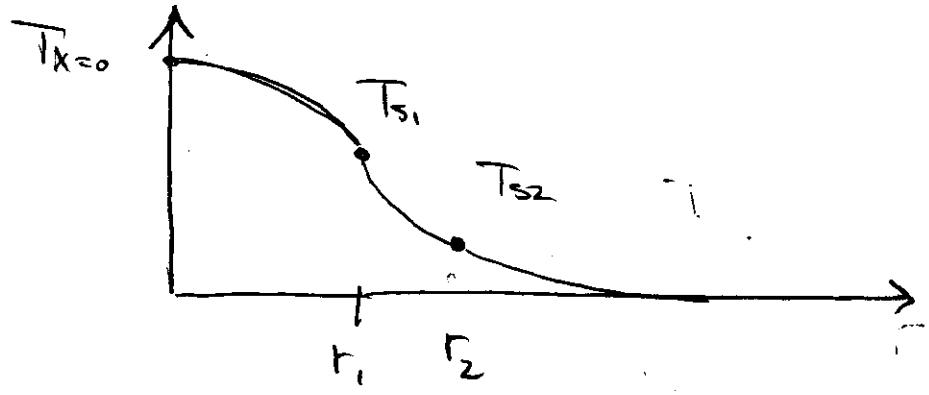
$$\frac{d}{dr}\left(k r \frac{dT}{dr}\right) = 0 \qquad \checkmark \text{ satisfied}$$

Verify $T(r) = T_{s_1} + \frac{\dot{q} r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right)$

$$\frac{dT}{dr} = \frac{\dot{q}\cancel{r_1^2}}{4k_c} \cdot -\frac{2r}{\cancel{r_1^2}}$$

$$k_c r \frac{dT}{dr} = \frac{-\dot{q}\cancel{k_c} r^2}{2\cancel{k_c}}$$

$$\frac{d}{dr}\left(k_c r \frac{dT}{dr}\right) = -\dot{q} \cdot \frac{\cancel{2}r}{\cancel{2}} \qquad \checkmark \text{ satisfied}$$



b) Applying Fouriers law (Insulation)

$$q''_r = -k \frac{\partial T}{\partial r}$$

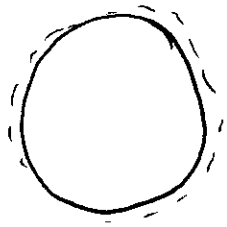
$$q''_r = -k \cdot \frac{1}{r} \frac{(T_{s_1} - T_{s_2})}{\ln(r_1/r_2)}$$

$$q' = 2\pi r \cdot q''$$

$$q'' = -2\pi k \frac{(T_{s_1} - T_{s_2})}{\ln(r_1/r_2)}$$

$$q'' = \frac{2\pi k (T_{s_1} - T_{s_2})}{\ln(r_2/r_1)}$$

applying energy balance around the cable

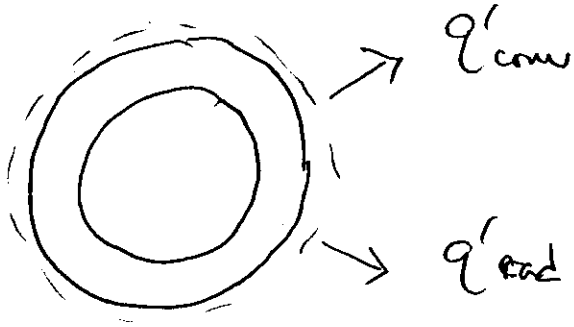


$$\dot{E}_{gen} = q_{out} = q' \cdot L$$

$$\dot{q} \cdot \pi r_1^2 \cdot L = q' \cdot L$$

$$\boxed{q' = \dot{q} \pi r_1^2}$$

c)



$$q' = \dot{q} \pi r_1^2 = q'_{conv} + q'_{rad}$$

$$\dot{q} \pi r_1^2 = h \cdot 2\pi r_2 \left(T_{s_2} - T_\infty\right) + \varepsilon\sigma \, 2\pi r_2 \left(T_{s_2}^4 - T_{surr}^4\right)$$

d) $I = 250\ A$

$R' = 0.005\ \Omega/m$

$r_1 = 15\ mm$ $\qquad k_c = 200\ W/m^2K$

$r_2 = 15.5\ mm$ $\qquad k_s = 0.15\ W/m^2K$

$h = 25\ W/m^2K$ $\qquad \varepsilon = 0.9$

$T_\infty = 25°C$ $\qquad T_{surr} = 35°C$

$$q' = I^2 R' = 312.5\ W/m$$

$$q' = h \cdot 2\pi r_2 (T_{s_2} - T_\infty) + \varepsilon\sigma\, 2\pi r_2 (T_{s_2}^4 - T_{surr}^4)$$

$$T_{s_2} = 395.0\ K$$

$$q' = \frac{2\pi k_s (T_{s_1} - T_{s_2})}{\ln(r_2/r_1)}$$

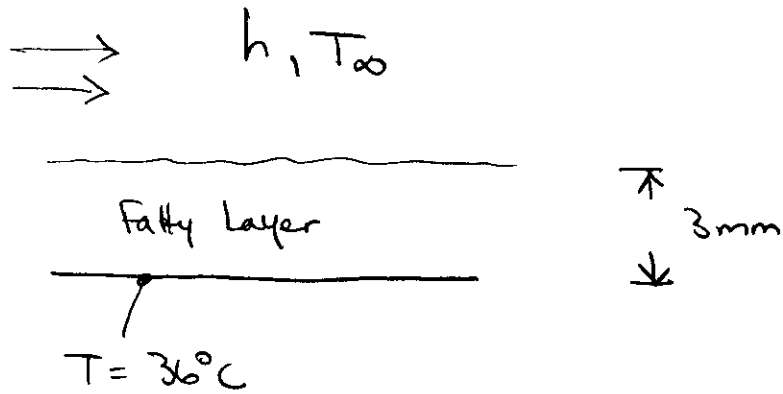
$$T_{s_1} = 405.9\ K$$

$$\dot{q} = \frac{q'}{\pi r_1^2} = 4.42 \times 10^{8}\ \frac{W}{m^3}$$

$$T(0) = T_{s_1} + \frac{\dot{q} r_1^2}{4k_c} = T_{s_1} + 0.1243$$

$$T(0) = 406.0\ K$$

Problem 3.7 Wind Chill



$K_f = 0.2\ W/mK$
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$h_{calm} = 25\ W/m^2K$

$h_{windy} = 65\ W/m^2K$

$T_\infty = -15°C$

$q'' = \frac{\Delta T}{R''}$

a) $R''_{calm} = \frac{L_f}{K_f} + \frac{1}{h_c} = 0.055\ \frac{m^2K}{W}$

$R''_{windy} = \frac{L_f}{K_f} + \frac{1}{h_w} = 0.0304\ \frac{m^2K}{W}$

$$\frac{q''_{windy}}{q''_{calm}} = \frac{R''_{calm}}{R''_{windy}} = \frac{L_f/K_f + 1/h_c}{L_f/K_f + 1/h_w} =$$

$$\boxed{\frac{q''_{windy}}{q''_{calm}} = 1.8}$$

b) Find Skin outside surface temperature

$$q''_{calm} = \frac{\Delta T}{R''_{calm}} = \frac{[36 - (-15)]\,K}{0.055\ \frac{m^2K}{W}} = 927.3\ \frac{W}{m^2}$$

$$q''_{calm} = h_{calm}\left[T_s - T_\infty\right]$$

$$T_s = \frac{q''}{h_{calm}} + T_\infty = 22.1\,°C$$

$$q''_{windy} = \frac{\Delta T}{R''_{windy}} = \frac{\left[36-(-15)\right]}{0.0304\ \frac{m^2 k}{W}} = 1677\ \frac{W}{m^2}$$

$$T_s = \frac{q''}{h_{windy}} + T_\infty = 10.8\,°C$$

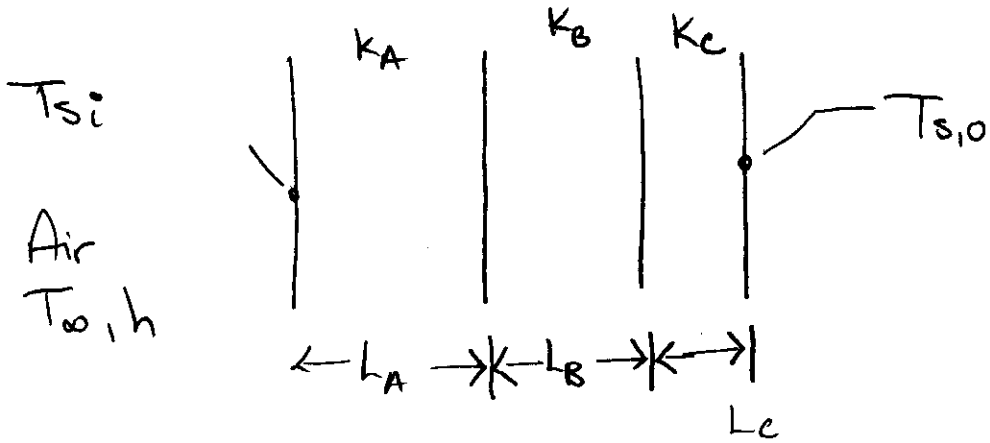
c) What temperature would be required to produce the same heat loss

$$q''_{calm} = 1677\ W/m^2 = \frac{\Delta T}{R''_{calm}}$$

$$\Delta T = 92.235$$

$$\boxed{T_\infty = -56.2\ °C}$$

Problem 3.9 Composite wall of Oven



$K_A = 20 \ \frac{W}{mK}$ $\qquad L_A = 0.3 \ m$

$K_C = 50 \ W/mK$ $\qquad L_C = 0.15 \ m$

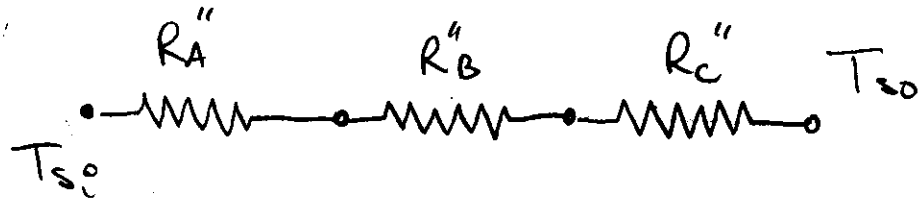
$K_B = ?$ $\qquad L_B = 0.15 \ m$

Given

$T_{si} = 600°C$ $\qquad h = 25 \ W/m^2K$

$T_\infty = 800°C$ $\qquad T_{s,o} = 20°C$

$$q'' = h\left[T_\infty - T_{si}\right] = 5000 \ W/m^2$$



$$R''_{WALL} = \frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C}$$

$$q'' = \frac{\Delta T}{R''} \qquad R''_{wall} = \frac{T_{si} - T_{so}}{q''}$$

$$R''_{WALL} = 0.1160 = \frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C}$$

Solve K_B

$$\frac{L_B}{K_B} = 0.098 \ \frac{m^2 K}{W}$$

$$K_B = 1.53 \ W/mK$$